Partial Fraction Decomposition Example:

$$\frac{x+2}{x^2(x+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+1)} + \frac{D}{(x+1)^2}$$

$$= \frac{A}{x} \cdot \frac{x(x+1)^2}{x(x+1)^2} + \frac{B}{x^2} \cdot \frac{(x+1)^2}{(x+1)^2} + \frac{C}{(x+1)} \cdot \frac{x^2(x+1)}{x^2(x+1)} + \frac{D}{(x+1)^2} \cdot \frac{x^2}{x^2}$$

$$= \frac{Ax(x+1)^2}{x^2(x+1)^2} + \frac{B(x+1)^2}{x^2(x+1)^2} + \frac{Cx^2(x+1)}{x^2(x+1)^2} + \frac{Dx^2}{x^2(x+1)^2}$$

$$= \frac{Ax(x+1)^2 + B(x+1)^2 + Cx^2(x+1) + Dx^2}{x^2(x+1)^2}$$

$$= \frac{Ax(x^2+2x+1) + B(x^2+2x+1) + Cx^3 + Cx^2 + Dx^2}{x^2(x+1)^2}$$

$$= \frac{Ax^3 + 2Ax^2 + Ax + Bx^2 + 2Bx + B + Cx^3 + Cx^2 + Dx^2}{x^2(x+1)^2}$$

$$= \frac{(A+C)x^3 + (2A+B+C+D)x^2 + (A+2B)x + B}{x^2(x+1)^2}$$

We are trying to find *A*, *B*, *C*, and *D* such that

$$0x^3 + 0x^2 + 1x + 2 = (A+C)x^3 + (2A+B+C+D)x^2 + (A+2B)x + B$$

Thus, **(i)** $A + C = 0$; **(ii)** $2A + B + C + D = 0$; **(iii)** $A + 2B = 1$; and **(iv)** $B = 2$.

Thus, we have a system with 4 equations. Since $B = 2$, equation **(iii)** gives us $A = -3$.

Substituting -3 for *A* into equation **(i)** gives us that $C = 3$.

Finally, substituting -3 for *A*, 2 for *B*, and 3 for *C* into equation **(ii)** gives us that $D = 1$.

Thus,

$$\frac{x+2}{x^2(x+1)^2} = \frac{-3}{x} + \frac{2}{x^2} + \frac{3}{(x+1)} + \frac{1}{(x+1)^2}$$

Although solving for *A*, *B*, *C*, and *D* in this case was a relatively short process, sometimes it may involve more steps. In these instances, it is often convenient to use matrix algebra. Our equations above can be written as follows:

i) $A + 0B + C + 0D = 0$
ii) $2A + B + C + D = 0$
iii) $A + 2B + 0C + 0D = 1$
iv) $0A + B + 0C + 0D = 2$

Which can be translated into the following matrix:

$$\left|\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \end{array}\right|$$

Then, using the rref function under [2nd] [Matrix] [MATH] on the calculator, we can reduce the matrix.

$$\left|\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right|$$

Thus, $A = -3$, $B = 2$, $C = 3$, and $D = 1$. So, we get the same solution as we did before.

Partial Fraction Decomposition Set Up Examples:

$$\frac{x+1}{x(x+1)} = \frac{A}{x} + \frac{B}{(x+1)}$$

$$\frac{x+2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{(x^2+1)}$$

$$\frac{x+3}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+1)}$$

$$\frac{x+4}{x^2(x+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+1)} + \frac{D}{(x+1)^2}$$

$$\frac{x+5}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{(x^2+1)} + \frac{Dx+E}{(x^2+1)^2}$$

$$\frac{x+6}{x^2(x^2+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{(x^2+1)} + \frac{Ex+F}{(x^2+1)^2}$$

$$\frac{x+7}{(x+1)^2(x^2+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{Cx+D}{(x^2+1)} + \frac{Ex+F}{(x^2+1)^2}$$

$$\frac{x+8}{(x+1)^3} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$