Gauss-Jordan Elimination

General Steps for Using Gauss-Jordan

Completing Column 1

$\_\_\_\_ \cdot R_1 \rightarrow R_1$ $\quad \left|\begin{array}{ccc|c} 1 & & & \\ & & & \\ & & & \end{array}\right|$

$\_\_\_\_ \cdot R_1 + R_2 \rightarrow R_2$

$\_\_\_\_ \cdot R_1 + R_3 \rightarrow R_3$ $\quad \left|\begin{array}{ccc|c} 1 & & & \\ 0 & & & \\ 0 & & & \end{array}\right|$

Completing Column 2:

$\_\_\_\_ \cdot R_2 \rightarrow R_2$ $\quad \left|\begin{array}{ccc|c} 1 & & & \\ 0 & 1 & & \\ 0 & & & \end{array}\right|$

$\_\_\_\_ \cdot R_2 + R_1 \rightarrow R_1$

$\_\_\_\_ \cdot R_2 + R_3 \rightarrow R_3$ $\quad \left|\begin{array}{ccc|c} 1 & 0 & & \\ 0 & 1 & & \\ 0 & 0 & & \end{array}\right|$

Completing Column 3:

$\_\_\_\_ \cdot R_3 \rightarrow R_3$ $\quad \left|\begin{array}{ccc|c} 1 & 0 & & \\ 0 & 1 & & \\ 0 & 0 & 1 & \end{array}\right|$

$\_\_\_\_ \cdot R_3 + R_1 \rightarrow R_1$

$\_\_\_\_ \cdot R_3 + R_2 \rightarrow R_2$ $\quad \left|\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \end{array}\right|$

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Tips for using Gauss-Jordan:

- To get 1's, use reciprocals
- To get 0's, use additive inverses
- Use column 1 to get zeroes in row 1.
- Use column 2 to get zeroes in row 2, and so on.

Three Possible Outcomes (Examples):

1) One Solution:

$$\left|\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 4 \end{array}\right|$$

$x = 5$, $y = 2$, and $z = 4$. The solution is (5, 2, 4).

2) Infinite Solutions (dependent system):

$$\left|\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & -3 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right|$$

The equations are:
$x + 2z = 3$ and $y - 3z = 4$

Solve for x and y:
$x = 3 - 2z$ and $y = 4 + 3z$

Thus, the solution is:
$(3 - 2z, 4 + 3z, z)$ with z = any real number

3) No Solution (inconsistent system):

$$\left|\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 4 \end{array}\right|$$

Here, this matrix tells us that 0 = 4 which we know is false. Thus, there is no solution.

Example:

Solve the system represented by the matrix: $\left|\begin{array}{ccc|c} 3 & 4 & 2 & 1 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{array}\right|$

The first step is to get a 1 in the first row of the first column.

Since there is a 1 in the second row, we'll do a row swap.

$\boldsymbol{rowSwap}([\boldsymbol{A}], \mathbf{1}, \mathbf{2}) \to [\boldsymbol{A}]$

$$R_1 \xleftrightarrow{SWAP} R_2$$

$$\left|\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \\ 2 & 3 & 1 & 4 \end{array}\right|$$

The second step is to get zeros in the remaining cells of the first column.

We will do this by using additive inverses.

$*\,\boldsymbol{row} + (-\mathbf{3}, [\boldsymbol{A}], \mathbf{1}, \mathbf{2}) \to [\boldsymbol{A}]$

$$-3R_1 + R_2 \xrightarrow{YIELDS\ THE\ NEW} R_2$$

$$\left|\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -2 & -7 & -11 \\ 2 & 3 & 1 & 4 \end{array}\right|$$

$*\,\boldsymbol{row} + (-\mathbf{2}, [\boldsymbol{A}], \mathbf{1}, \mathbf{3}) \to [\boldsymbol{A}]$

$$-2R_1 + R_3 \xrightarrow{YIELDS\ THE\ NEW} R_3$$

$$\left|\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -2 & -7 & -11 \\ 0 & -1 & -5 & -4 \end{array}\right|$$

Now that we are done with the first column, we move to the second column. The next step is to get a 1 in the second row of the second column.

$*\,\boldsymbol{row}(-\,\mathbf{1/2}\,, [\boldsymbol{A}], \mathbf{2}) \to [\boldsymbol{A}]$

$$-\frac{1}{2}R_2 \xleftrightarrow{YIELDS\ THE\ NEW} R_2$$

$$\left|\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 7/2 & 11/2 \\ 0 & -1 & -5 & -4 \end{array}\right|$$

We now get zeros in the remaining entries of column 2.

$\boldsymbol{row} + ([\boldsymbol{A}], \mathbf{2}, \mathbf{3}) \to [\boldsymbol{A}]$

$$R_2 + R_3 \xrightarrow{YIELDS\ THE\ NEW} R_3$$

$$\left|\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 7/2 & 11/2 \\ 0 & 0 & -3/2 & 3/2 \end{array}\right|$$

$*\,\boldsymbol{row} + (-\mathbf{2}, [\boldsymbol{A}], \mathbf{2}, \mathbf{1}) \to [\boldsymbol{A}]$

$$-2R_2 + R_1 \xrightarrow{YIELDS\ THE\ NEW} R_1$$

$$\left|\begin{array}{ccc|c} 1 & 0 & -4 & -7 \\ 0 & 1 & 7/2 & 11/2 \\ 0 & 0 & -3/2 & 3/2 \end{array}\right|$$

Now that we are done with the second column, we move to the third column. We begin by getting a 1 in the third row. $*\,\boldsymbol{row}(-\,\mathbf{2/3}\,, [\boldsymbol{A}], \mathbf{3}) \to [\boldsymbol{A}]$

$$-\frac{2}{3}R_3 \xleftrightarrow{YIELDS\ THE\ NEW} R_3$$

$$\left|\begin{array}{ccc|c} 1 & 0 & -4 & -7 \\ 0 & 1 & 7/2 & 11/2 \\ 0 & 0 & 1 & -1 \end{array}\right|$$

Finally, we get zeros in the remaining entries of column 3.

$*\,\boldsymbol{row} + (-\,\mathbf{7/2}\,, [\boldsymbol{A}], \mathbf{3}, \mathbf{2}) \to [\boldsymbol{A}]$

$$-\frac{7}{2}R_3 + R_2 \xrightarrow{YIELDS\ THE\ NEW} R_2$$

$$\left|\begin{array}{ccc|c} 1 & 0 & -4 & -7 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & -1 \end{array}\right|$$

$*\,\boldsymbol{row} + (\mathbf{4}, [\boldsymbol{A}], \mathbf{3}, \mathbf{1}) \to [\boldsymbol{A}]$

$$4R_3 + R_1 \xrightarrow{YIELDS\ THE\ NEW} R_1$$

$$\left|\begin{array}{ccc|c} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & 9 \\ 0 & 0 & 1 & -1 \end{array}\right|$$